A General Equilibrium Model of Housing and Land

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Upzoning affects housing prices by changing the distribution of land across zoning types. For example, upzoning for apartments increases the supply of apartment-zoned land and reduces the supply of house-zoned land; or, expanding the urban growth boundary increases the supply of residential land and reduces the supply of agricultural land. To understand upzoning, we need a supply and demand model with both land and housing. In particular, we need a *general* equilibrium model, where both the price of land and the price of housing are determined within the model. By comparison, a *partial* equilibrium model takes one of the prices as fixed, and determined outside of the model.

In this note, I write down a simple general equilibrium model and derive a core result: increasing the supply of land reduces the land price, which shifts the supply of housing and reduces the housing price. So upzoning lowers housing prices by reducing the cost of high-density land. Specifically, we model a world of single-family houses built on agricultural land. There is an urban growth boundary (UGB) that constrains the housing supply. I model upzoning as an expansion in the UGB, which increases the supply of residential land by reallocating parcels from agricultural to residential. In general, upzoning reallocates land from low- to high-density, so this model should be informative for upzoning from detached houses to apartments.

1 Setup

Consider a one-time build-out of a new city of single-family houses on identical undeveloped agricultural land. Each house requires one parcel of land, so the quantity of houses equals the quantity of parcels. Let Q denote this quantity. There are $\bar{Q}$ parcels available to build on, due to an urban growth boundary limiting the size of the city. To show the effect of upzoning, we increase the UGB and allow more land to be built on. When the UGB is a constraint on housing supply, housing prices P are bid up above the cost of land and construction, and land prices r are bid up above the agricultural land rent $\bar{r}$. Expanding the UGB directly increases the supply of housing and the supply of residential land, by relaxing the zoning constraint.[1] Hence, UGB expansion leads to lower land and housing prices.

There are three agents:

1. **Landowners.** Assume that initially all land is homogeneous agricultural land: all landowners sell at the same price, and the supply of land is perfectly elastic up to the UGB.[2] If they don't sell, landowners earn the agricultural rent $\bar{r}$.

[1] Note that relaxing a binding constraint is distinct from reducing the input cost of land, e.g., through improved agricultural productivity reducing demand for farmland and hence lowering $\bar{r}$. Relaxing a constraint increases production of the output directly, whereas reducing an input cost shifts the input supply curve, which in turn shifts the output supply curve.

[2] An upward-sloping supply curve requires heterogeneity in reservation prices, for example, from differences in the value of existing use or teardown costs.

2. **Developers.** Developers buy parcels of land and build single-family houses. Since one parcel is needed per house, developers choose a single quantity Q.[3] Under perfect competition, profits are zero, so a developer's marginal revenue is equal to their marginal cost: they sell a house for P and pay $r + c$ in costs, for constant marginal construction cost c. Hence, $P = r + c$.

3. **Homebuyers.** Demand for houses slopes downward in the house price.[4]

The model is:

$$r = \bar{r} + \lambda, \quad 0 \leq Q \leq \bar{Q}, \quad \lambda \geq 0, \quad \lambda(\bar{Q} - Q) = 0, \tag{1}$$

$$P = r + c, \tag{2}$$

$$P = y - zQ. \tag{3}$$

Equation (1) is the inverse supply curve for land. Here $\bar{r} > 0$ is the rental price of agricultural land, and λ is the shadow value of the constraint, i.e., the marginal value of relaxing the constraint. Either $\lambda > 0$ and $Q = \bar{Q}$ (the constraint is binding) or $\lambda = 0$ and $Q < \bar{Q}$ (the constraint is slack). Equation (2) is the developer's zero-profit condition, from which we can derive the residual land value $r = P - c$. Marginal construction costs are constant through $c > 0$. Equation (3) is the inverse demand curve for housing, with slope $z > 0$, so $1/z$ is the slope of direct demand for housing.

From (2) and (3), we get the inverse demand function for land:

$$r = (y - c) - zQ. \tag{4}$$

So if $z > 0$, the demand curve for land is downward-sloping, and increasing supply reduces its price. This is the *general equilibrium* mechanism, where downward-sloping demand for housing (via (3)) generates downward-sloping demand for land. Because houses and land are linked, additional quantity of land is used to produce more housing, which (when $z > 0$) decreases P, which in turn reduces developers' willingness-to-pay for land. Hence, the slope of the demand curve for land is structurally determined by the model. If demand for housing is perfectly elastic, then $P = \bar{P}$ and the demand for land is also perfectly elastic: $r = \bar{P} - c$.

From (2) and (1), we get the inverse supply function for housing:

$$P = \begin{cases} c + \bar{r}, & Q < \bar{Q} \\ c + \bar{r} + \lambda, & Q = \bar{Q}. \end{cases} \tag{5}$$

Below the constraint, the price of housing is the marginal cost of construction plus the cost of land. When the land constraint binds, the scarcity premium λ increases housing prices.[5]

The inverse supply function for land is also defined piecewise:

$$r = \begin{cases} \bar{r}, & Q < \bar{Q} \\ \bar{r} + \lambda, & Q = \bar{Q}. \end{cases} \tag{6}$$

Also note that perfectly elastic supply assumes infintely many sellers, which is not true here. So the model is assuming away market power. We could account for price-setting by using Nash bargaining between landowners and developers.

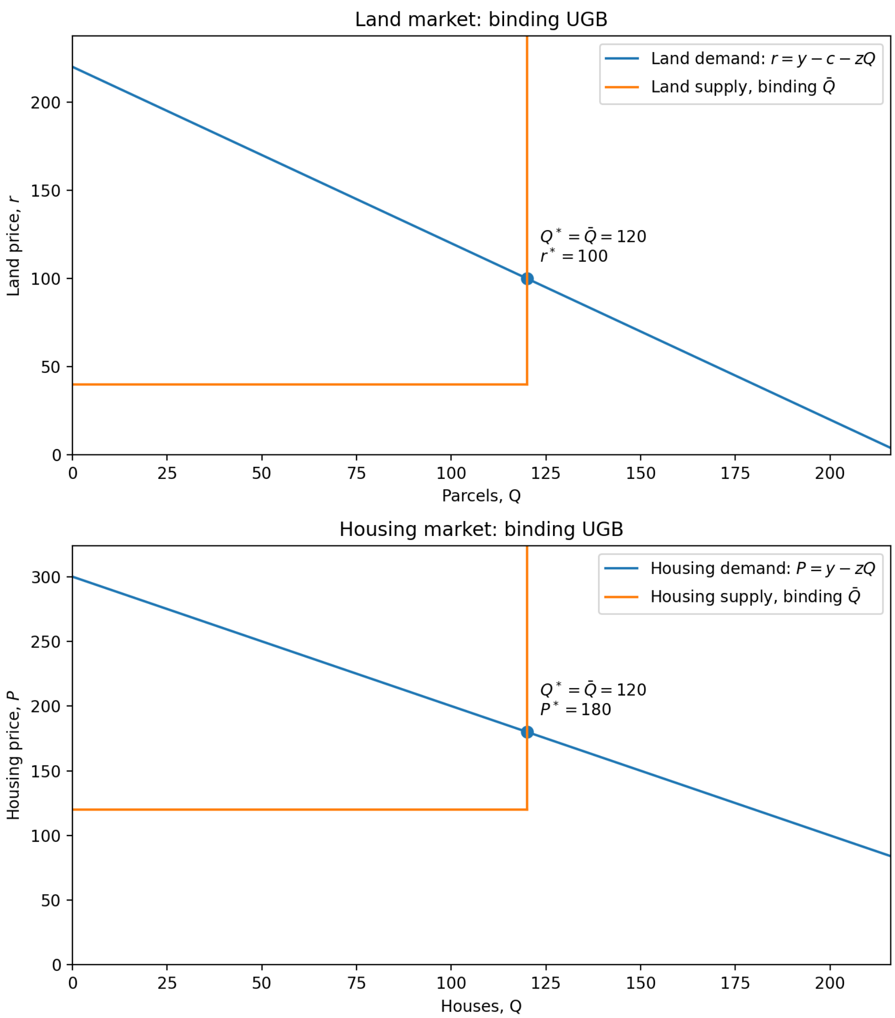
[3] In the general case, developers can build multiple homes per parcel, so that $Q_H^S = \alpha Q_L^D$: each home requires a fraction α of a parcel. This model uses $\alpha = 1$.

[4] Note that this is a one-location model: all parcels are identical, so commuting costs are constant.

[5] This is identical to the zoning tax in Glaeser and Gyourko (2003): in their setup, $P = K + pL + T$, for housing price P, construction cost K, free market land price p, land units L, and zoning tax T. The mapping is: $c = K$, $\bar{r} = p$, $L = 1$, and $T = \lambda$.

Hence, below the constraint, both the supply of land and the supply of housing are perfectly elastic: $r = \bar{r}$ and $P = c + \bar{r}$. That is, the supply functions are horizontal up to $\bar{Q}$, then becomes vertical at the constraint.[6]

Figure 1: Binding UGB equilibrium in land and housing markets



Parameters: $y = 300, z = 1, c = 80, \bar{r} = 40, \bar{Q} = 120$.

[6] Technically, the supply function becomes a supply relation at $\bar{Q}$, since it fails the vertical line test. So $r = \bar{r} + \lambda$ is the supply curve evaluated at the equilibrium.

2 UGB is not binding

In the unconstrained case where the UGB is not binding, we have the interior solution with unconstrained quantity $Q_U < \bar{Q}$:

$$r^* = \bar{r}, \tag{7}$$
$$P^* = c + \bar{r}, \tag{8}$$
$$Q^* = Q_U = \frac{y - c - \bar{r}}{z}. \tag{9}$$

Here the land price is determined within the land market, rather than as a residual, because it is not a fixed factor.[7]

3 UGB is a binding constraint

When the UGB is a binding constraint ($Q_U > \bar{Q}$, i.e., the unconstrained equilibrium quantity of land is larger than the capacity constraint), then we have the corner solution:

$$Q^* = \bar{Q}, \tag{10}$$
$$P^* = y - z\bar{Q}, \tag{11}$$
$$r^* = (y - c) - z\bar{Q}. \tag{12}$$

In this case, land is a fixed factor, so its price is determined as a residual ($r^* = P^* - c$). The scarcity premium is

$$\lambda = r^* - \bar{r} = (y - c - \bar{r}) - z\bar{Q}. \tag{13}$$

Intuitively, residential land becomes more expensive than the agricultural use by amount λ when residential capacity is artificially scarce. So when capacity binds, the difference between the land price and the agricultural rent is exactly the marginal value of relaxing the constraint. Note that λ is exactly the Glaeser and Gyourko regulatory tax.[8]

4 Comparative statics: expanding the UGB

UGB expansion is an increase in $\bar{Q}$, converting agricultural land to residential. In the unconstrained case, the equilibrium values do not depend on $\bar{Q}$, so increasing $\bar{Q}$ has no effect. In the constrained case, we get these comparative statics:

$$\frac{\partial Q^*}{\partial \bar{Q}} = 1 > 0, \quad \frac{\partial P^*}{\partial \bar{Q}} = \frac{\partial r^*}{\partial \bar{Q}} = \frac{\partial \lambda}{\partial \bar{Q}} = -z < 0. \tag{14}$$

So relaxing the constraint increases the quantity of land and housing, and reduces the price of each. The effect on r and the scarcity premium λ is the same, since r just is the agricultural rent plus the scarcity premium. So as λ shrinks to zero, the constraint stops binding, and further increases in $\bar{Q}$ have no effect. Hence we converge to the unconstrained equilibrium with $r = \bar{r}$, where

[7] Could other prices be determined as a residual? Yes, if they are a fixed factor. Suppose c represents construction labor; if a union restricts labor supply ($L = \bar{L}$), then $Q = \bar{L}$ and wages would be determined residually as $c = P^* - r^*$. If multiple factors are fixed, then the residual would be split by bargaining power.

[8] The regulatory tax is the price of housing minus construction costs minus the free-market price of land. This is $P - c - \bar{r} = r - \bar{r} = \lambda$.

residential and agricultural land have the same price. This suggests an intuitive policy goal: high- and low-density land should have the same price, and this occurs when zoning is not a constraint ($\lambda = 0$).

The *own-parcel* effect on the 'upzoned' parcel is the increase in value from the agricultural rent $\bar{r}$ to the new residential price r^{new}.[9] For $\Delta\bar{Q} = 1$ (expanding the UGB by one parcel), this is an increase from $\bar{r}$ to $r^{\text{new}} = (y - c) - z(\bar{Q} + 1)$. Generally, the own-parcel effect is the increase in option value from reallocating a parcel from a lower-density zone to a higher-density zone.

The *cross-parcel* effect on existing residential land is the drop in price from $r^{\text{initial}} = (y-c)-z\bar{Q}$ to $r^{\text{new}} = (y - c) - z(\bar{Q} + 1)$. This occurs because upzoning increases the number of residential parcels. Generally, the cross-parcel effect is the decrease in the higher-density land price driven by a larger number of parcels in the higher-density zone. Note that this effect is undefined for the first upzoning: when there are no higher-density parcels, the price of the higher-density zone is undefined.[10]

Hence, the upzoned parcel increases in value, but to a lower price than in the initial equilibrium. We can decompose the own-parcel effect into the *initial* own-parcel effect (before prices adjust) plus the cross-parcel effect: $r^{\text{new}} - \bar{r} = (r^{\text{initial}} - \bar{r}) + (r^{\text{new}} - r^{\text{initial}})$. This is due to the supply effect reducing the scarcity premium.[11]

And notice that downzoning has the expected opposite effects. The downzoned parcel has a negative own-parcel effect, since it switches from residential to agricultural and loses option value. The remaining residential land has a positive cross-parcel effect, since residential land becomes scarcer, raising housing prices and increasing residual land values. (If all land is downzoned, then there is no remaining residential land to experience the cross-parcel effect.)

How does the transition across equilibria work? Intuitively, increasing $\bar{Q}$ reduces r because at the initial price, there is excess supply of land, so the price falls to clear the land market. With lower r, developers produce more houses; since demand for housing is unchanged, P must fall to clear the housing market.[12][13]

Note that this is an example of 'partial upzoning'. We are increasing the stock of residential land by one additional unit; this allows us to ask how adding one parcel of residential land affects the price of residential land. This is the cross-parcel effect: upzoning one additional parcel reduces the price of existing upzoned land. Existing models of upzoning instead capture 'total upzoning', where all low-density land is simultaneously converted to high-density.[14] Since there is no existing

[9] We assume that converting some agricultural land to residential does not affect $\bar{r}$. That is, $\bar{r}$ is exogenous, so the model is partial equilibrium with respect to the low-density category (agricultural land).

[10] We get the cross-parcel effect because we upzone one additional parcel ('partial upzoning'). Models that upzone all land simultaneously ('total upzoning') miss this effect, since r^{initial} is undefined when there are no residential parcels before upzoning.

[11] When using the residual land value formula and taking housing prices as fixed, we miss this effect. This is assuming $z = 0$, i.e., perfectly elastic demand for housing. Recall that, at the constraint, the demand function for land is $r = y - c - z\bar{Q}$. So if $z > 0$, increasing $\bar{Q}$ reduces r.

[12] Since r and P are jointly determined in general equilibrium, it's a mistake to think that r can change only if P changes first. The residual land value formula is accounting, not economics. As noted above, the reason UGB expansion decreases r is because downward-sloping demand for housing from homebuyers leads to downward-sloping demand for land from developers.

[13] Contrast this to the AMM argument (Greenaway-McGrevy et al. 2025): relaxing the density constraint allows developers to produce more floorspace; assuming imperfect mobility, at initial prices there are more households per acre, so the population-clearing condition doesn't hold: the city can fit the population N on a smaller area. So the housing price must fall to maintain market clearing, which decreases the residual land value.

Hence, the AMM argument starts from a fall in housing prices, whereas my model starts from a fall in land prices. Note: AMM is about intensity of land use for a fixed stock of land, where all land at distance x is upzoned (total upzoning); my model is about the quantity of residential land, and captures partial upzoning.

[14] The standard AMM models a uniform height restriction at distance x, so relaxing the restriction affects all parcels

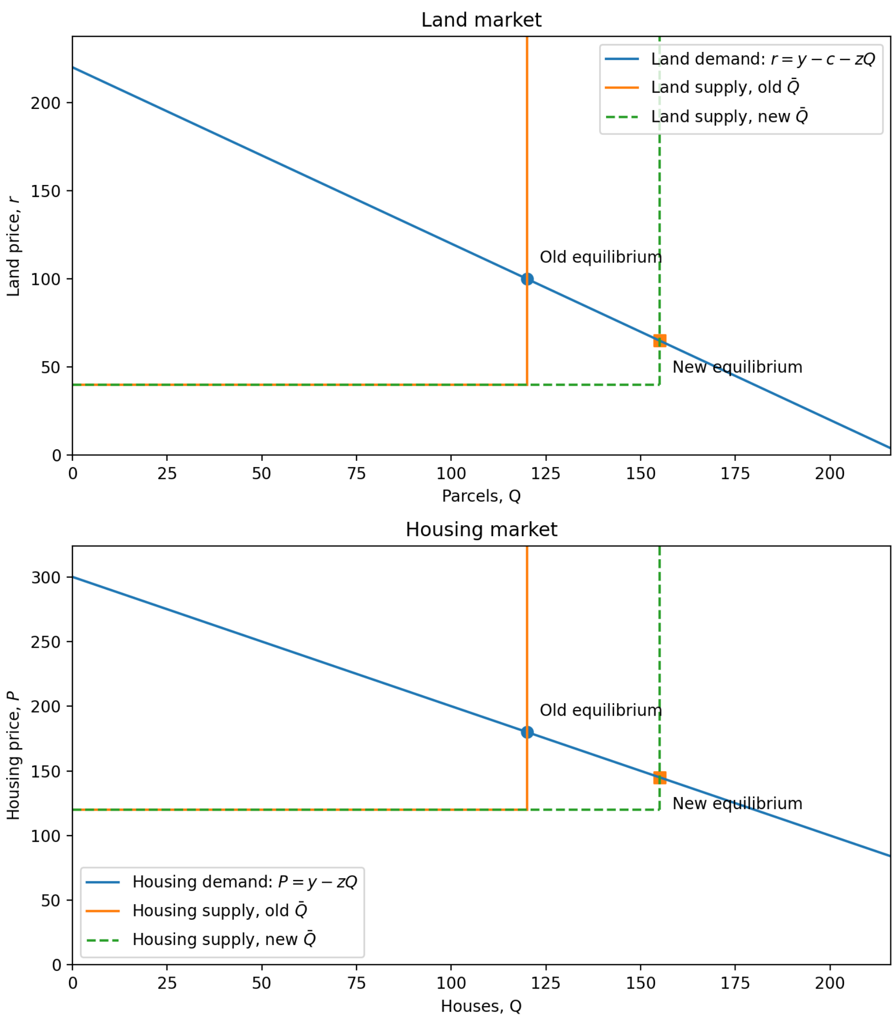
high-density land prior to upzoning, the cross-parcel effect is undefined (e.g., r^{initial} is not defined when there are zero residential parcels prior to upzoning). Instead, these models capture only the own-parcel effect, i.e., the increase in value from $\bar{r}$ to r^{new}.

at that location. Diamond et al. (2025) and Kulka et al. (2025) model the case where all land is upzoned.

4.1 Graphs

In the binding case, UGB expansion shifts the supply of land to the right. This increases the quantity of land and houses, and decreases the price of both land and housing. Here the initial equilibrium is: $r = 100, P = 180, Q = 120$. The new equilibrium is: $r = 65, P = 145, Q = 155$. The own-parcel effect is $r^{\text{new}} - \bar{r} = 65 - 40 = 25$, and the cross-parcel effect is $r^{\text{new}} - r^{\text{old}} = 65 - 100 = -35$. The regulatory tax ($\lambda = r - \bar{r}$) is initially 60 and falls to 25.

Figure 2: Comparative statics: UGB expansion in land and housing markets



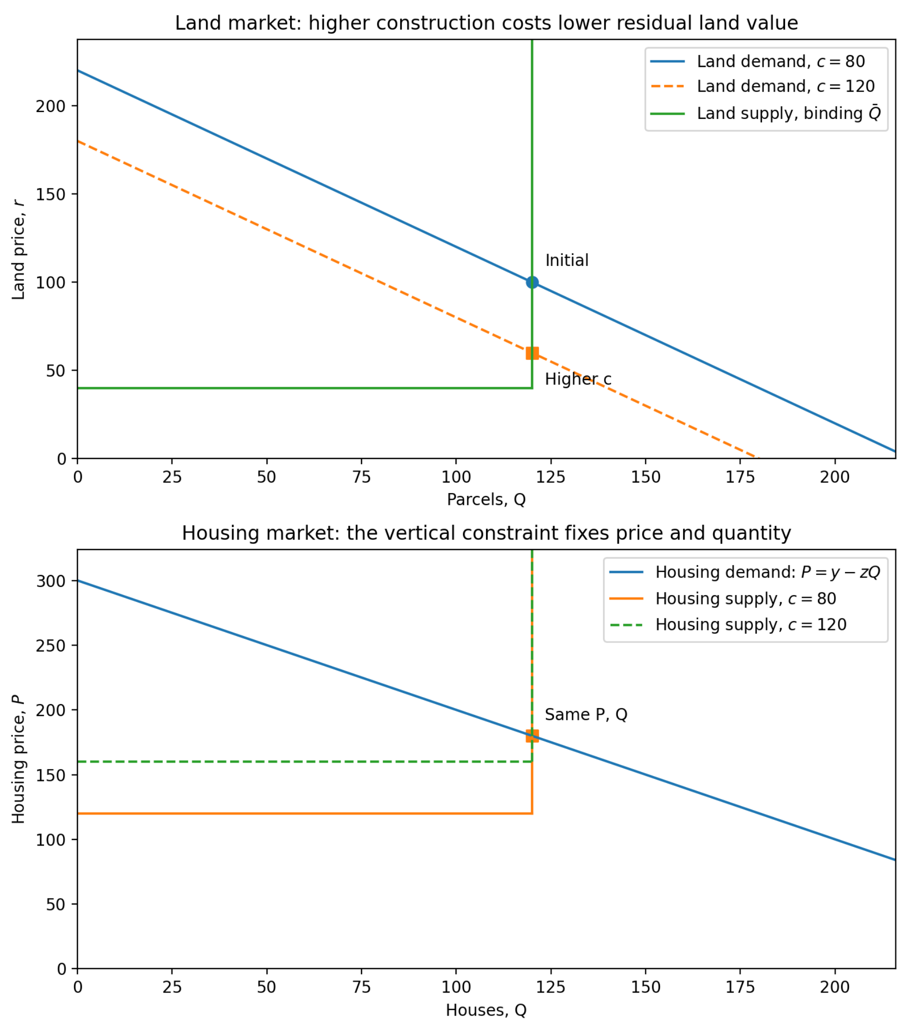
Parameters: $y = 300, z = 1, c = 80, \bar{r} = 40, \bar{Q}_{\text{old}} = 120, \bar{Q}_{\text{new}} = 155$.

5 Do higher construction costs increase housing prices?

When the UGB is slack and supply and demand intersect on the horizontal segment of the inverse supply curve (defined by $P = r + c$), we have an interior solution with equilibrium price $P^* = c + \bar{r}$. Here, higher construction costs (increased c) lead to higher housing prices.

But if the UGB is binding and demand intersects the vertical segment of the supply curve, then c can increase without affecting housing prices. Intuitively, prices are set by land scarcity, not construction costs. This is because $P^* = y - z\bar{Q}$, so if the increase in c is small enough that the UGB remains binding, housing prices are unchanged. Graphically, the housing supply curve shifts up, but this doesn't change the equilibrium. Land prices do fall, because $r^* = y - c - z\bar{Q}$ and the land demand curve shifts down.

Figure 3: Comparative statics: construction cost increase in land and housing markets



Parameters: $y = 300, z = 1, c_0 = 80, c_1 = 120, \bar{r} = 40, \bar{Q} = 120$.

6 Tax incidence of development charges

Suppose a per-unit development charge $\tau \geq 0$ is levied on developers at the land transaction. The statutory incidence is on developers, but the economic incidence falls on homebuyers or landowners. The developer's zero-profit condition is

$$P = r + c + \tau, \tag{15}$$

and the inverse demand for land is

$$r = P - c - \tau = (y - c - \tau) - zQ. \tag{16}$$

Since statutory incidence falls on developers, the inverse supply of land ($r = \bar{r} + \lambda$) and inverse demand for housing ($P = y - zQ$) are unchanged. The inverse supply of housing is now

$$P = c + \bar{r} + \lambda + \tau, \tag{17}$$

so the wedge between price and cost now includes the scarcity premium and the tax.

6.1 UGB is slack

In the unconstrained case, we have the equilibrium

$$r^* = \bar{r}, \tag{18}$$
$$P^* = c + \bar{r} + \tau, \tag{19}$$
$$Q^* = Q_U = \frac{y - c - \bar{r} - \tau}{z}. \tag{20}$$

So the comparative statics with respect to the tax are

$$\frac{\partial r^*}{\partial \tau} = 0, \frac{\partial P^*}{\partial \tau} = 1, \frac{\partial Q^*}{\partial \tau} = -\frac{1}{z}. \tag{21}$$

Since landowners are perfectly elastic, the incidence is fully on homebuyers, and the tax increases price and reduces quantity. Here, taxes raise housing prices. Graphically, the demand curve for land shifts down and the supply curve for housing shifts up (while the housing demand curve and land supply curve are unchanged).

6.2 UGB is binding

When the constraint binds, landowners are perfectly inelastic and the tax comes out of the scarcity premium. The equilibrium is

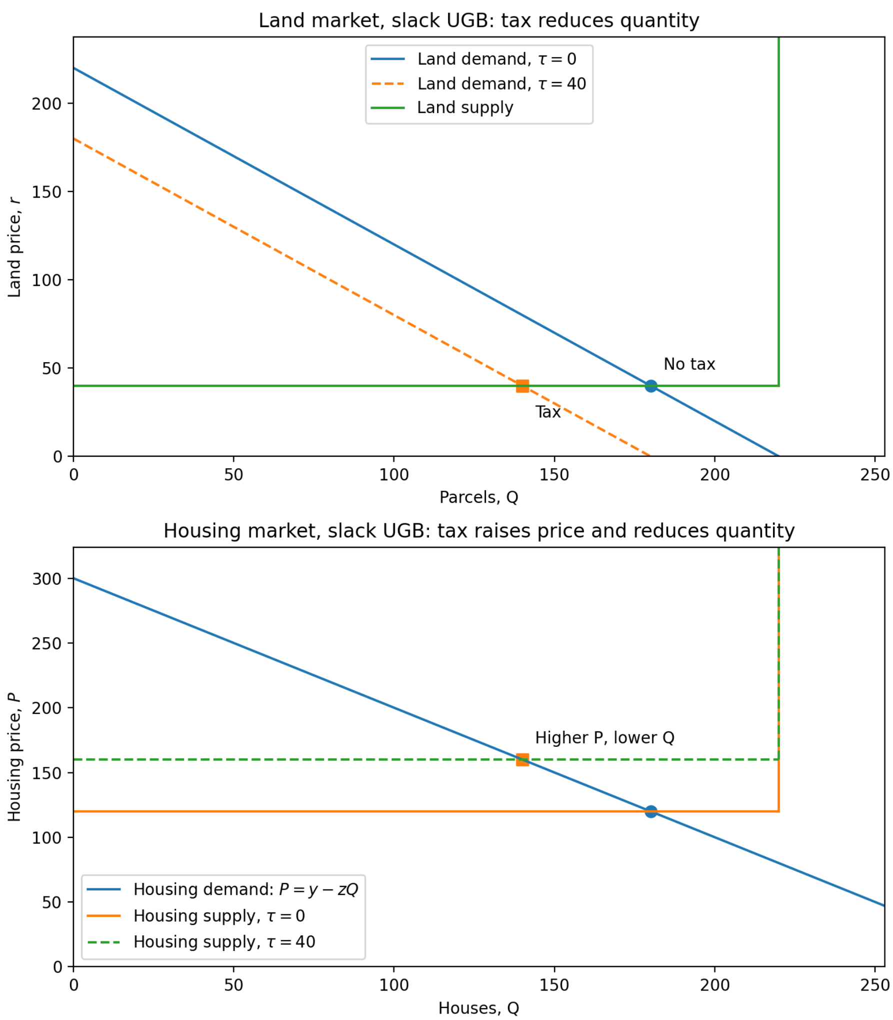
$$Q^* = \bar{Q}, \tag{22}$$
$$P^* = y - z\bar{Q}, \tag{23}$$
$$r^* = (y - c - \tau) - z\bar{Q}, \tag{24}$$
$$\lambda^* = r^* - \bar{r} = y - c - \bar{r} - \tau - z\bar{Q}. \tag{25}$$

The comparative statics are

$$\frac{\partial r^*}{\partial \tau} = \frac{\partial \lambda^*}{\partial \tau} = -1, \frac{\partial P^*}{\partial \tau} = \frac{\partial Q^*}{\partial \tau} = 0, \tag{26}$$

When the UGB is binding, the tax comes entirely out of land values, reducing the scarcity premium. It has no effect on the price or quantity of housing.

Figure 4: Comparative statics: development charge with slack UGB



Parameters: $y = 300, z = 1, c = 80, \bar{r} = 40, \bar{Q} = 220, \tau_0 = 0, \tau_1 = 40.$

6.3 Discussion

With no tax, the scarcity premium is $\lambda_0 = y - c - \bar{r} - z\bar{Q}$. Imposing a tax reduces the scarcity premium to $\lambda = \lambda_0 - \tau$. So when $\tau \leq \lambda_0$, the tax incidence is fully on landowners. (Here the tax is calibrated.) For $\tau > \lambda_0$, the UGB no longer binds and we are back in the slack case, where further increases in τ are paid by homebuyers. Graphically, if we decrease land demand enough that it intersects the horizontal segment of the supply curve, then the constraint is not binding.

So the general tax incidence is:

$$\frac{\partial P^*}{\partial \tau} = \begin{cases} 0, \tau < \lambda_0, \\ 1, \tau > \lambda_0 \end{cases} \qquad \frac{\partial r^*}{\partial \tau} = \begin{cases} -1, \tau < \lambda_0, \\ 0, \tau > \lambda_0. \end{cases} \tag{27}$$

Here tax incidence is either 100% on landowners or homebuyers. If the land supply curve is upward-sloping due to heterogeneous reservation values, then we would get continuous incidence based on the pass-through fraction. Note that there's no incidence on developers here, because developers are homogeneous. If we allow developers to have increasing marginal costs, then there would be some incidence on developers.

Should the government impose an UGB in order to tax the scarcity premium? No, this is perverse. Restricting the supply of residential land raises housing prices by creating a scarcity premium. Taxing the scarcity premium merely reallocates it from landowners to the government. Even if the tax revenue is used to fund subsidized housing, overall housing affordability is worsened; in the best case scenario, housing prices would fall back to the initial level. We cannot improve affordability by first creating scarcity.

Similarly, if the government is dependent on revenue from development charges, then upzoning by removing the UGB reduces tax revenue by decreasing housing prices. Here, the government has a vested interest in maintaining scarcity.

We can interpret the tax as value capture, i.e., taxing land lift from upzoning. Initially $\bar{Q} = 0$ and no development is allowed, so all land is priced at $\bar{r}$; then we expand the UGB, so upzoned land values increase by $\lambda = r^* - \bar{r}$. (Since there's no existing high-density land, there's no cross-parcel effect.) Hence, we're taxing the surplus land value created by upzoning. For the initial equilibrium above, this land lift is $\lambda = 100 - 40 = 60$.

Note that if we upzone to $\bar{Q} = 180$, land prices fall to $\bar{r}$ and land lift is zero. Mass upzoning eliminates the scarcity premium, so there is nothing for the government to tax.[15] The surplus goes to homebuyers in the form of lower housing prices. So if the government cares more about tax revenue than housing affordability, it will restrict supply to maintain a scarcity premium that it can tax. Conversely, a government that cares about aggregate social welfare will upzone as much as possible to minimize housing prices.

As with construction costs, imposing a tax shifts the demand for land down and shifts the supply for housing up. In the binding UGB case, this reduces land prices with no effect on housing prices.

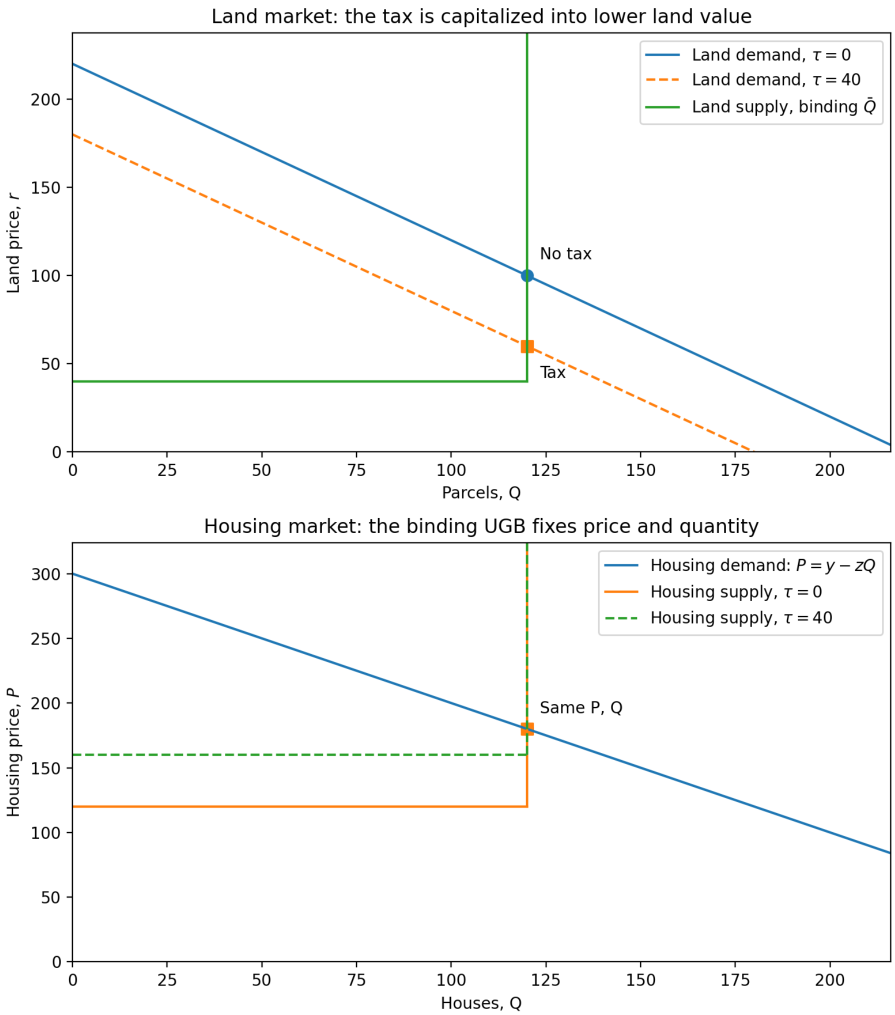
Figure 5 shows an increase in development charges, assessed on the land transaction. Note that r is the producer price of land (received by landowners), so rewriting the land demand function in terms of the producer price shifts it down; the price received by landowners falls. We could also use the consumer price of land (paid by developers); here we rewrite the land supply function in terms of the consumer price, which shifts it up by τ, with no change in the price (including τ) paid by developers.

[15] If land is inherently limited (for example, desirable locations), then we can hit a non-zoning supply constraint before eliminating the scarcity premium. In this case, there is a non-zero scarcity premium we can tax, even under mass upzoning.

Since the tax is assessed on the land market, the housing market has no wedge between the price paid and received. Here, the housing supply function shifts up due to higher after-tax costs, but the housing price is unchanged. (Hence, the distinction between consumer vs producer price does not matter here.)

If the tax is assessed on the housing transaction, then the housing market has a wedge and the land market does not. Using the consumer price paid by homebuyers, the housing supply curve shifts up; homebuyers pay the same price. Using the producer price received by developers, the housing demand curve shifts down; developers receive the consumer price minus the tax. In the land market, there is no wedge. The tax shifts the land demand curve down, which holds using either consumer or producer price. So developers pay $r - \tau$ for land, which is what landowners receive.

Figure 5: Comparative statics: tax increase in land and housing markets



Parameters: $y = 300, z = 1, c = 80, \bar{r} = 40, \bar{Q} = 120, \tau_0 = 0, \tau_1 = 40.$

7 Per-parcel land value tax

An unconditional land value tax on both residential and agricultural land has no effect on housing prices or quantities. A per-parcel LVT of size T reduces the landowner's agricultural outside option: $r = \bar{r} - T$. Hence, the land supply curve is shifted down by T:

$$r = \bar{r} - T + \lambda, \quad 0 \leq Q \leq \bar{Q}, \quad \lambda \geq 0, \quad \lambda(\bar{Q} - Q) = 0. \tag{28}$$

The LVT also reduces the developer's payoff from residential land, so the zero-profit condition is:

$$P - T = r + c \iff P = r + c + T. \tag{29}$$

Combining with the demand curve for housing, this shifts down the land demand curve:

$$r = y - c - T - zQ. \tag{30}$$

Since both curves shift down, the LVT has no effect on the equilibrium quantity of land.

Moreover, since the tax appears in both the land supply curve and the developer's zero-profit condition, it cancels out in the housing supply curve:

$$P = \bar{r} - T + \lambda + c + T = c + \bar{r} + \lambda. \tag{31}$$

Hence, the housing market is unaffected. Intuitively, an LVT does not change the margin between agricultural and residential land use, since both are taxed equally, so the equilibrium is unchanged. A developer has to pay T when they own residential land, but since agricultural land is now worth less, the land price is also reduced by T, and the net effect is a wash. (This is the key difference from a development charge, which applies conditionally to development, and hence does not shift the land supply function.)

Note that $T \leq \bar{r}$, or owning agricultural land has negative value. So in the slack case, we can tax at most $\bar{r}$, which means a parcel sells at a price of $r^* = 0$.

7.1 UGB is slack

In the unconstrained case, we have the equilibrium

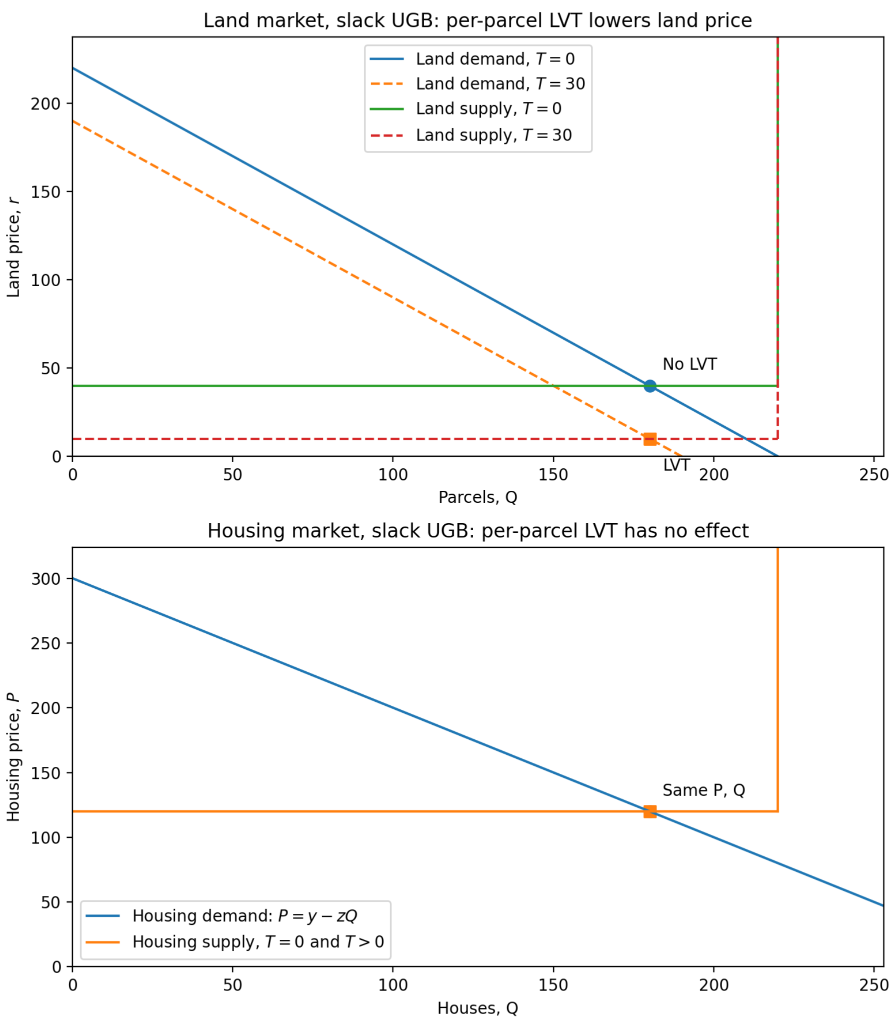
$$r^* = \bar{r} - T, \tag{32}$$
$$P^* = c + \bar{r}, \tag{33}$$
$$Q^* = Q_U = \frac{y - c - \bar{r}}{z}. \tag{34}$$

So the comparative statics with respect to the LVT are

$$\frac{\partial r^*}{\partial T} = -1, \frac{\partial P^*}{\partial T} = \frac{\partial Q^*}{\partial T} = 0. \tag{35}$$

The LVT reduces land prices, even though landowners are perfectly elastic producers of residential land; this is because the tax is applied to all land, not just residential land used for housing. There are no effects on the housing market.

Figure 6: Comparative statics: per-parcel LVT with slack UGB



Parameters: $y = 300, z = 1, c = 80, T = 30, \bar{r} = 40, \bar{Q} = 220$.

7.2 UGB is binding

In the constrained case, the equilibrium is

$$Q^* = \bar{Q}, \tag{36}$$

$$P^* = y - z\bar{Q}, \tag{37}$$

$$r^* = (y - c - T) - z\bar{Q}, \tag{38}$$

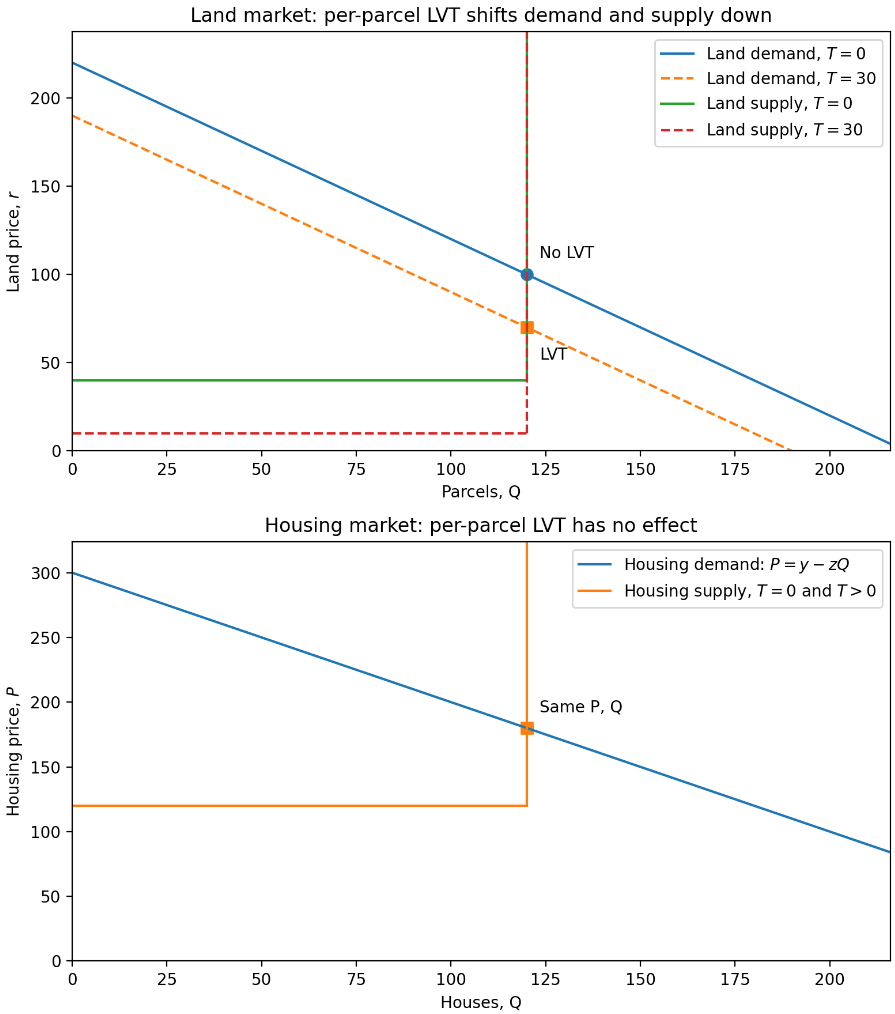
$$\lambda^* = r^* - (\bar{r} - T) = y - c - \bar{r} - z\bar{Q}. \tag{39}$$

The comparative statics are

$$\frac{\partial r^*}{\partial T} = -1, \frac{\partial P^*}{\partial T} = \frac{\partial Q^*}{\partial T} = \frac{\partial \lambda^*}{\partial T} = 0, \tag{40}$$

A 'specific' per-parcel tax does not affect the scarcity premium, since we are taxing all land by the same amount. But when zoning is a constraint, residential land becomes more valuable than agricultural land, so we need a proportional 'ad valorem' tax.

Figure 7: Comparative statics: per-parcel LVT in land and housing markets



Parameters: $y = 300, z = 1, c = 80, T = 30, \bar{r} = 40, \bar{Q} = 120$.

8 Proportional LVT

Let t be a proportional LVT assessed on the value of the land. This reduces the agricultural outside option from $\bar{r}$ to $(1-t)\bar{r}$. So the land supply curve is shifted down:

$$r = (1-t)\bar{r} + \lambda, \quad 0 \leq Q \leq \bar{Q}, \quad \lambda \geq 0, \quad \lambda(\bar{Q} - Q) = 0. \tag{41}$$

The LVT reduces how much the landowner receives from the developer. For residual land value $P - c$, the landowner receives a share $1 - t$. Hence, the developer zero-profit condition is:

$$(1-t)(P-c) = r \iff P = c + \frac{r}{1-t}. \tag{42}$$

Combining with the demand curve for housing, the land demand curve is shifted down:

$$r = (y - c - zQ)(1-t). \tag{43}$$

Both land curves shift down, cancelling any effect on the quantity of land.

The tax also cancels out in the housing supply curve:

$$P = c + \frac{(1-t)\bar{r} + \lambda}{1-t} = c + \bar{r} + \frac{\lambda}{1-t}. \tag{44}$$

So the housing market is unaffected. Intuitively, after-tax land costs are higher, but this is cancelled out by lower land prices. In this case, the LVT does reduce the scarcity premiums.

8.1 UGB is slack

In the unconstrained case, we have the equilibrium

$$r^* = (1-t)\bar{r}, \tag{45}$$

$$P^* = c + \bar{r}, \tag{46}$$

$$Q^* = Q_U = \frac{y - c - \bar{r}}{z}. \tag{47}$$

So the comparative statics with respect to the LVT are

$$\frac{\partial r^*}{\partial t} = -\bar{r}, \frac{\partial P^*}{\partial t} = \frac{\partial Q^*}{\partial t} = 0. \tag{48}$$

A proportional LVT reduces land prices proportionally.

8.2 UGB is binding

In the constrained case, the equilibrium is

$$Q^* = \bar{Q}, \tag{49}$$

$$P^* = y - z\bar{Q}, \tag{50}$$

$$r^* = (1-t)(y - c - z\bar{Q}), \tag{51}$$

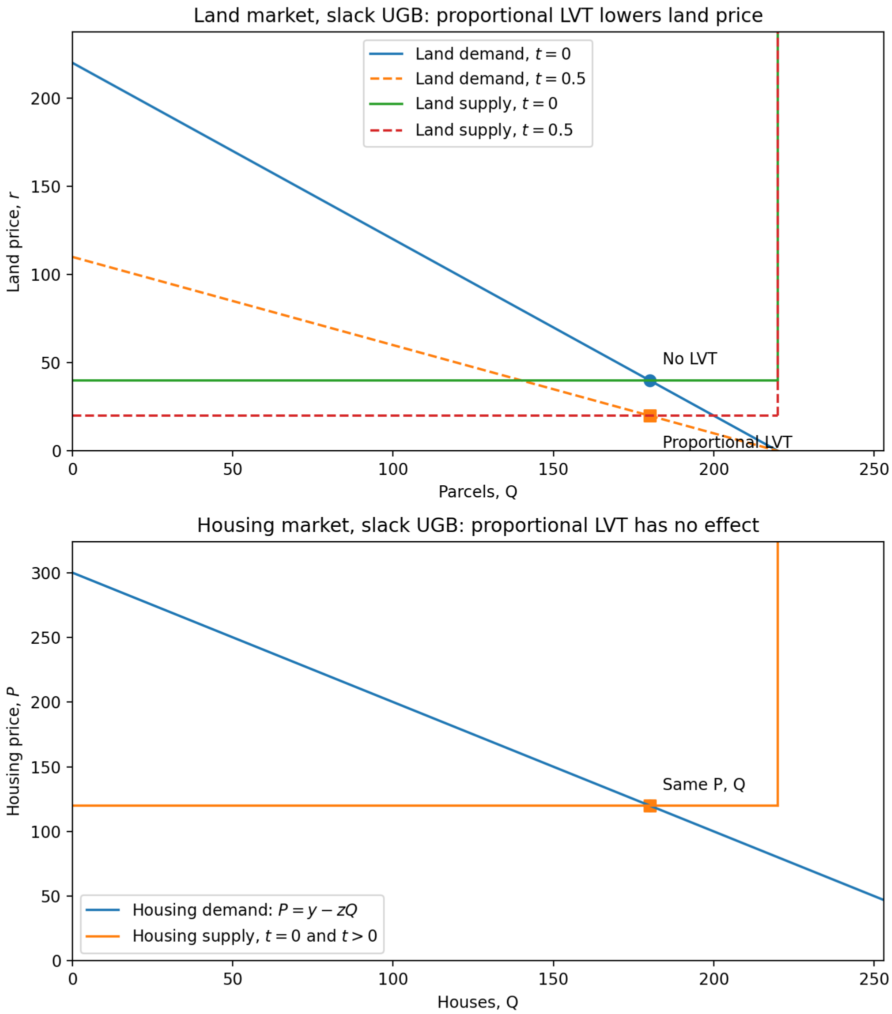
$$\lambda^* = r^* - (1-t)\bar{r} = (1-t)(y - c - \bar{r} - z\bar{Q}). \tag{52}$$

The comparative statics are

$$\frac{\partial r^*}{\partial t} = -(y - c - z\bar{Q}), \frac{\partial \lambda^*}{\partial t} = -(y - c - \bar{r} - z\bar{Q}), \frac{\partial P^*}{\partial t} = \frac{\partial Q^*}{\partial t} = 0. \tag{53}$$

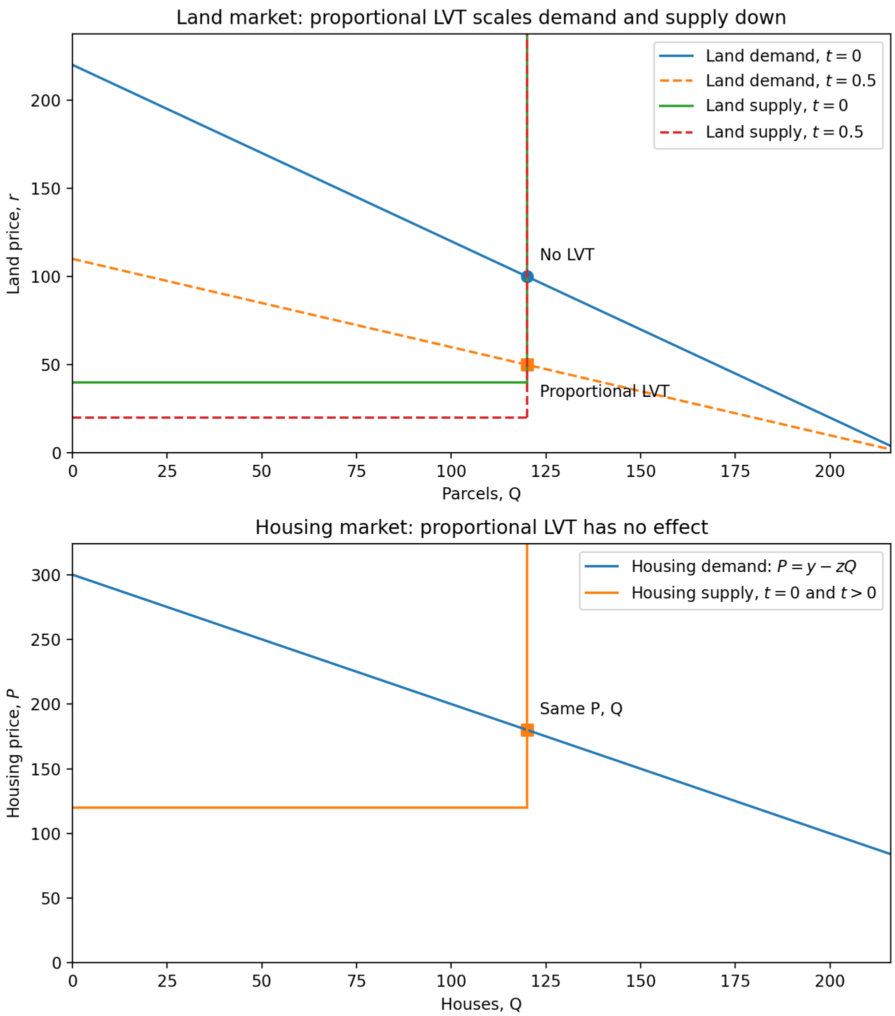
In the proportional case, an LVT reduces the scarcity premium. When $t = 1$, both the land price and scarcity premium are 0. Hence, LVT works for value capture.

Figure 8: Comparative statics: proportional LVT with slack UGB



Parameters: $y = 300, z = 1, c = 80, t = 0.5, \bar{r} = 40, \bar{Q} = 220.$

Figure 9: Comparative statics: proportional LVT in land and housing markets

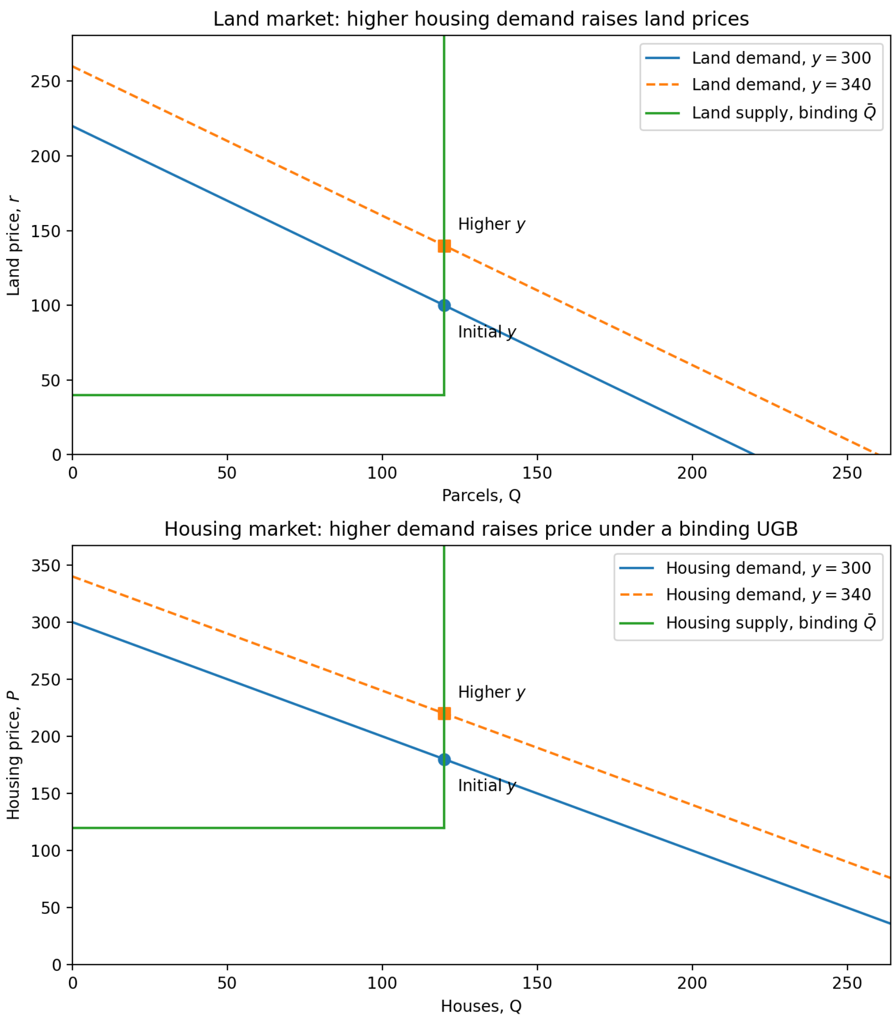


Parameters: $y = 300, z = 1, c = 80, t = 0.5, \bar{r} = 40, \bar{Q} = 120.$

9 Land prices are driven by housing demand

A common argument among non-economists is that upzoning drives up land prices. While it is true that upzoning can have a positive own-parcel effect, rising land prices are generally driven by increased housing demand. In the binding case, land prices are directly a function of the demand curve intercept (y): $r = y - c - z\bar{Q}$. Hence, increases in demand directly raise land prices. This is consistent with the natural experiment of First Shaughnessy, a heritage-protected mansion neighborhood in Vancouver, where assessed land values increased five-fold despite no upzonings. Since demand increased 5x and supply is restricted, land prices also increased 5x.

Figure 10: Comparative statics: higher housing demand with binding UGB

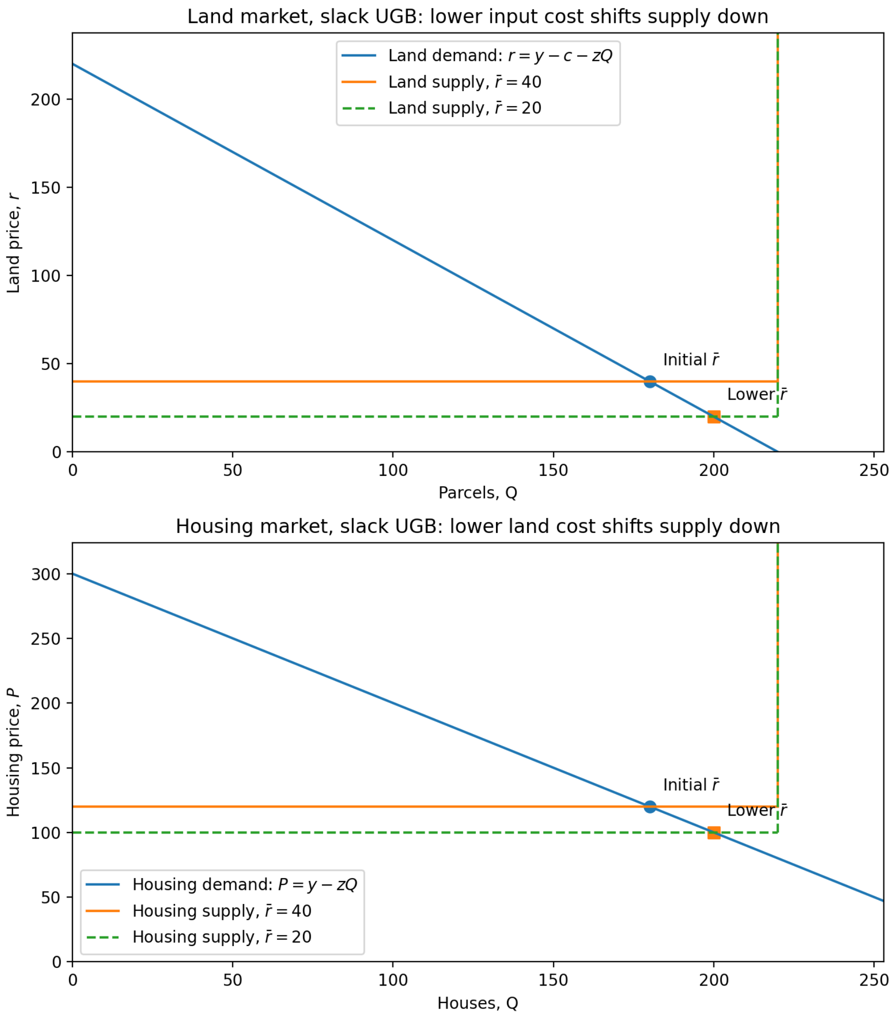


Parameters: $y_0 = 300, y_1 = 340, z = 1, c = 80, \bar{r} = 40, \bar{Q} = 120$.

10 Reducing land input costs

Relaxing a binding zoning constraint has a direct effect on housing supply; UGB expansion shifts both the supply of housing and land. A different way to increase supply of the output good is by reducing input costs. For example, improvements in productivity reduce demand for farmland, which reduces the agricultural rent $\bar{r}$. This shifts the supply of land $r = \bar{r} + \lambda$, which in turn shifts the supply of housing $P = c + r$. In this case, the effect on housing supply is indirect, whereas relaxing a constraint has a direct effect.

Figure 11: Comparative statics: reducing land input costs with slack UGB



Parameters: $y = 300, z = 1, c = 80, \bar{r}_0 = 40, \bar{r}_1 = 20, \bar{Q} = 220.$

References

Glaeser, E. and J. Gyourko (2003). The impact of building restrictions on housing affordability. *Federal Reserve Bank of New York Economic Policy Review 9*, 21–39.